

**MTH 211, Math for Architects, Exam II, Spring 2014**

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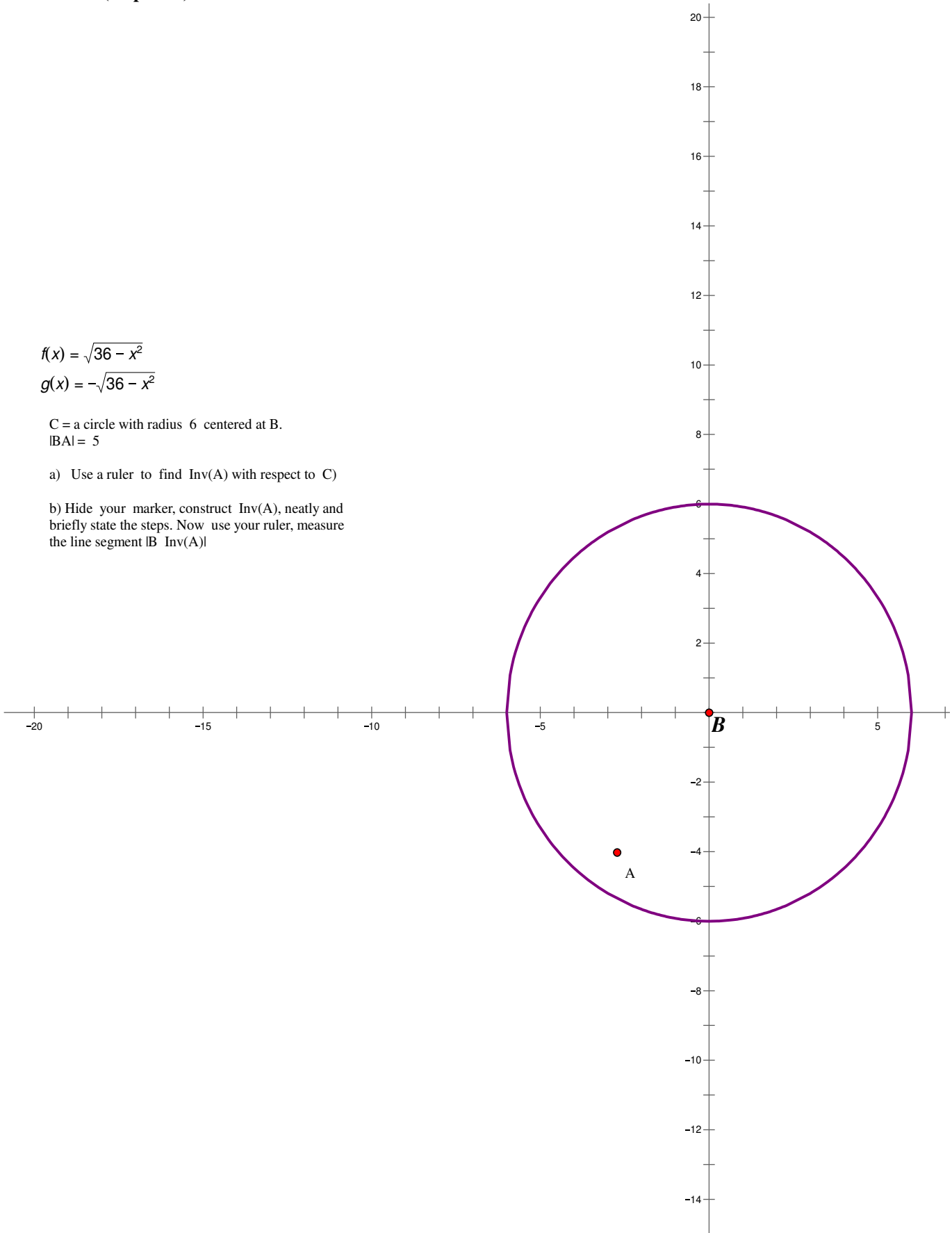
**QUESTION 1. (10 points).** Consider

$$f(x) = \sqrt{36 - x^2}$$

$$g(x) = -\sqrt{36 - x^2}$$

C = a circle with radius 6 centered at B.  
 $|BA| = 5$

- Use a ruler to find  $\text{Inv}(A)$  with respect to C
- Hide your marker, construct  $\text{Inv}(A)$ , neatly and briefly state the steps. Now use your ruler, measure the line segment  $|B \text{Inv}(A)|$



**QUESTION 2. (10 points).** Consider

$$f(x) = \sqrt{64 - x^2}$$

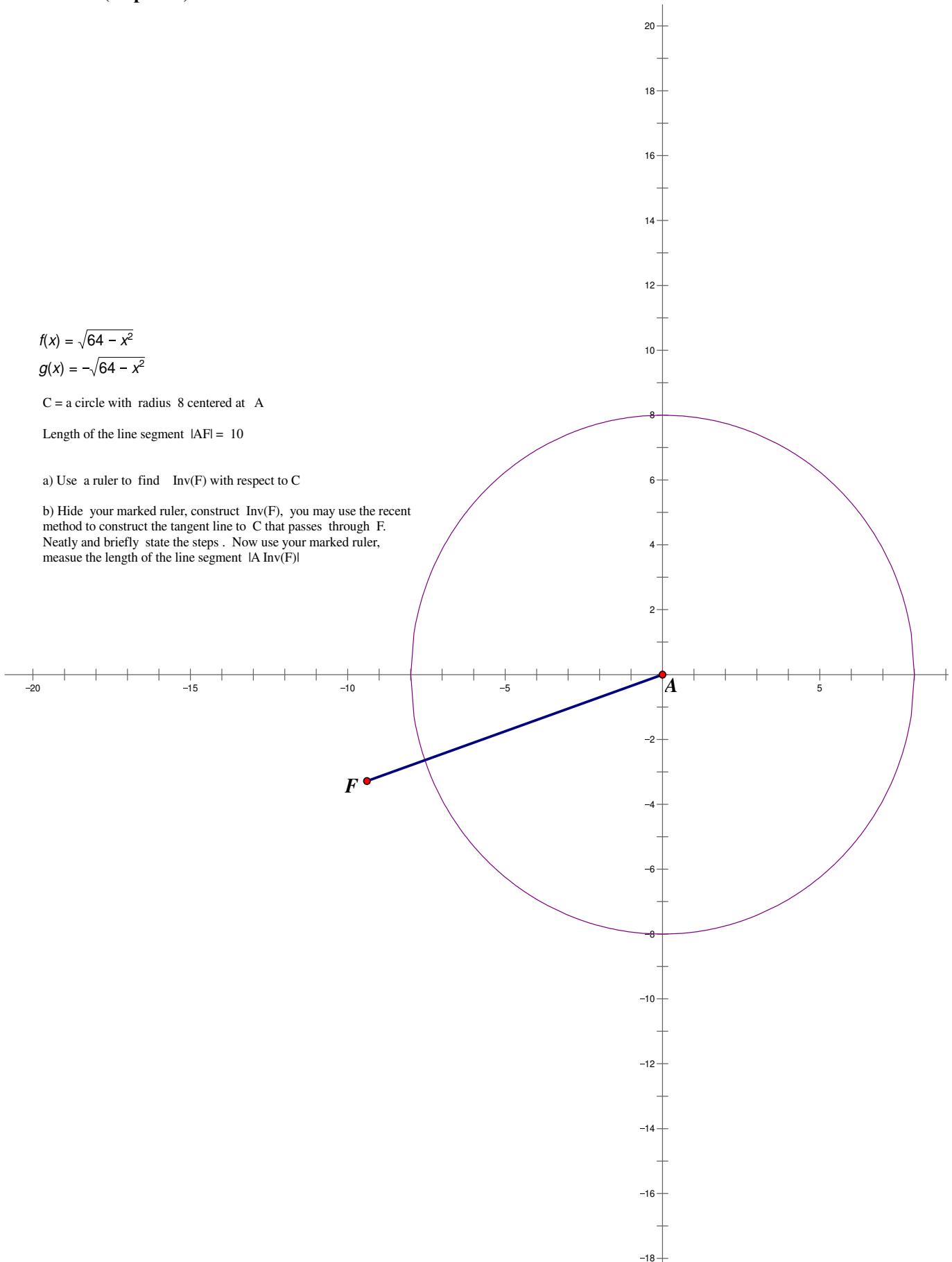
$$g(x) = -\sqrt{64 - x^2}$$

$C$  = a circle with radius 8 centered at  $A$

Length of the line segment  $|AF| = 10$

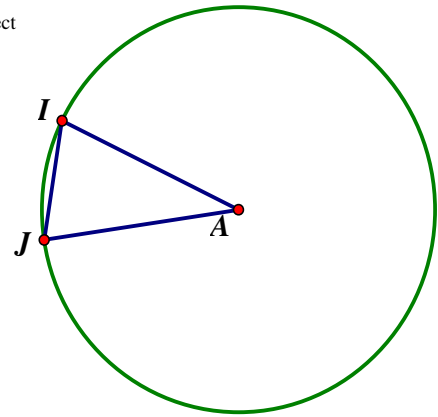
a) Use a ruler to find  $\text{Inv}(F)$  with respect to  $C$

b) Hide your marked ruler, construct  $\text{Inv}(F)$ , you may use the recent method to construct the tangent line to  $C$  that passes through  $F$ . Neatly and briefly state the steps. Now use your marked ruler, measure the length of the line segment  $|A \text{Inv}(F)|$

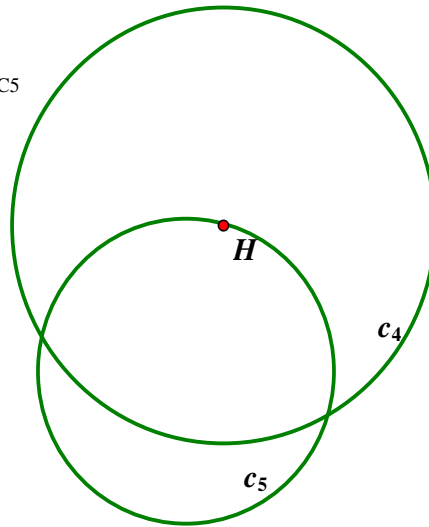


**QUESTION 3. (10 points).** Consider

$C$  is a circle centered at  $A$ . Find the general shape of the inversion of the triangle  $AIJ$  with respect to  $C$ . You don't need to do the actual (exact) inversion,



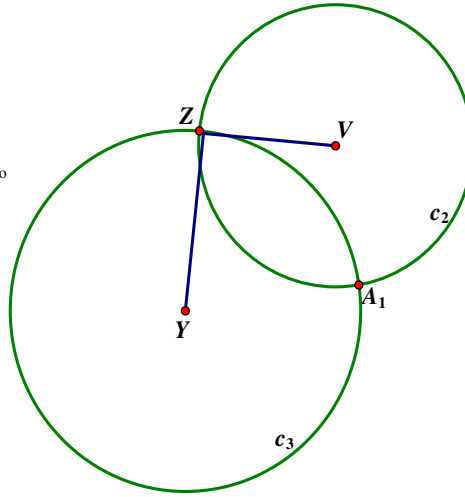
$C_4$  is a circle centered at  $H$ . Find the exact (the actual) inversion of  $C_5$  with respect to  $C_4$ .



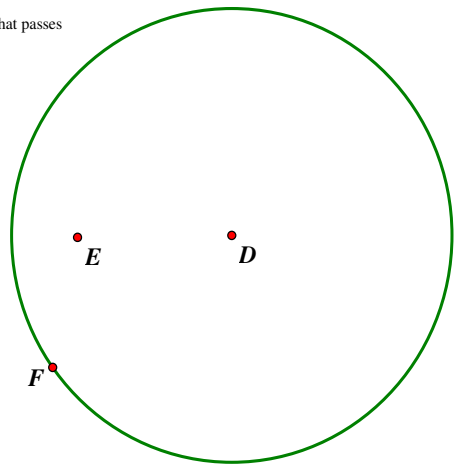
**QUESTION 4. (20 points).** Consider

Given  $ZV$  is perpendicular to  $ZY$  at  $Z$ . What is the inversion of the circle  $C_3$  with respect to  $C_2$ ? explain

What is the inversion of the arc  $ZA_1$  of the circle  $C_3$  that is inside  $C_2$  with respect to  $C_2$ ?



Given a circle  $C$  centered at  $D$ . State neatly and briefly the steps that you would follow in order to construct a circle  $M$  that passes through  $E$  and  $F$  such that  $M$  is orthogonal to  $C$ .



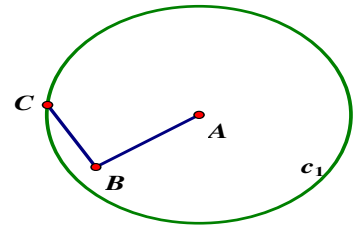
For the non-Euclidean hyperbolic geometry, answer the following::

- 1) Sum of the interior angles of any triangle is always .....
- 2) If  $Q$  is a point not on a line  $L$ , how many lines are there passing through  $Q$  and parallel to  $L$ ?
- 3) If  $Q$  is a real point and  $B$  is a horizon point, then what is the maximum number of lines that are passing through  $Q$  and  $B$ ?

**QUESTION 5. (15 points).** Consider

$C_1$  is a circle with radius 5 centered at  $A$ ,  $CB$  is perpendicular to  $AB$  at  $B$  and assume  $|CB| = 3$ .

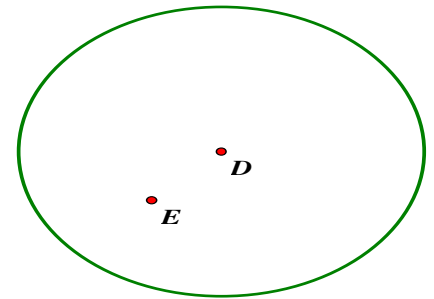
1) Find the length of the line segment  $A\text{Inv}(B)$ , i.e., find  $|A\text{Inv}(B)|$ .



$C$  is a circle with radius 4 centered at  $D$ .  $|DE| = 2$ . Let  $L$  be a circle passes through  $E$  and orthogonal to  $C$ .

1) What is the smallest radius of  $L$ ?

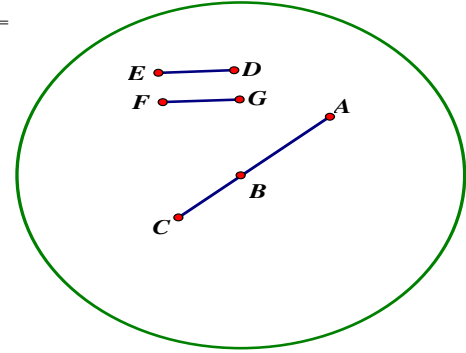
2) Can we construct such  $L$  with radius  $\sqrt{13}$ ? If yes construct such  $L$  with radius EXACTLY  $\sqrt{13}$



**QUESTION 6. (15 points).** Consider

Given the Hyperbolic circle  $H$  with radius  $6$  centered at  $B$ . Given  $C, B, A$  lie on the same line segment  $AC$ ,  $d(A, B) = 4$ ,  $d(C, B) = 2$ .

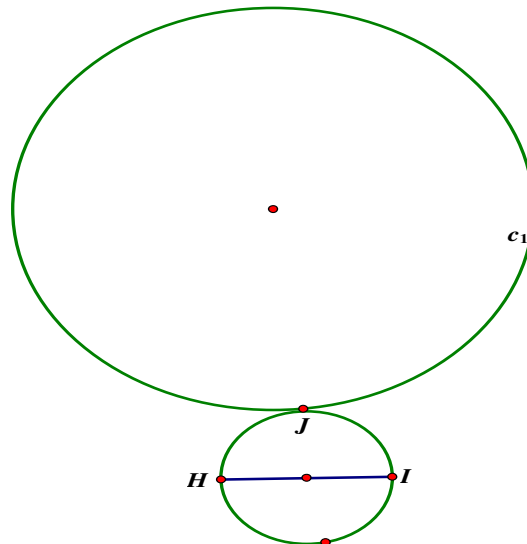
1) Find the hyperbolic distance between  $A$  and  $C$ , i.e. find  $d_h(A, C)$



2) Given  $d(E, D) = d(F, G)$  (see picture). Can we conclude that  $d_h(E, D) = d_h(F, G)$ ?  $d_h(E, D) < d_h(F, G)$ ?  $d_h(E, D) > d_h(F, G)$ ? briefly Explain your conclusion.

Find the inversion of  $HIJ$  with respect to  $C_1$ . Just draw the general shape of the inversion (it need not be exact).

Note that  $HIJ$  consists of the line segment  $HI$  and the upperhalf ARC.

**Faculty information**

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